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THE MACH NUMBER DEPENDENCE OF THE STAGNATION POINT HEAT TRANSFER IN SUPERSONIC FLOW

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NOMENCLATURE

- a, velocity of sound;
- C, Chapman's constant;
- F, F_1, functions defined by equations (6) and (4) respectively;
- h, film coefficient;
- k, heat conductivity coefficient;
- M, Mach number;
- m, exponent in the Falkner–Skan type of external velocity distribution, $u_e = c \,.\, x^m$;
- Nu, Nusselt number;
- p, pressure;
- Pr, Prandtl number;
- q, heat flux per unit area and time;
- R, body radius, sphere or cylinder;
- Re, Reynolds number;
- S, enthalpy function, $S = \dfrac{H}{H_e} - 1$, H total enthalpy;
- T, temperature;
- u, velocity in the x-direction in the physical plane;
- U, velocity in the X-direction in the transformed plane (Stewartson transformation);
- x, streamwise distance in the physical plane;
- X, streamwise distance in the transformed plane (Stewartson transformation);
- y, distance normal to the wall in the physical plane.

Greek symbols

- β, pressure gradient parameter defined by equation (3);
- γ, specific heat ratio;
- θ, angle between stream direction and radius vector from the center of curvature of the nose;
- μ, ν, dynamic and kinematic viscosity respectively;
- ρ, density.

Subscripts

- ∞, upstream infinity conditions;
- 0, stagnation conditions;
- e, conditions at the outer edge of the boundary layer;
- w, wall conditions;
- 1, values evaluated immediately downstream of the normal shock.

IN THIS note a simplified expression for the Mach number dependence of the film coefficient at the stagnation point of spheres and cylinders in supersonic flow is presented.

Assuming a perfect gas, the Nusselt number can be written:

$$Nu_\infty = -\frac{h\,.\,R}{k_\infty} = -\frac{q_w\,.\,R}{k_\infty(T_0 - T_w)} = \frac{R\,.\left(k\dfrac{\partial T}{\partial y}\right)_w}{k_\infty(T_0 - T_w)} \tag{1}$$

with $Pr = 1{\cdot}0$.

The temperature gradient normal to the surface for both the two-dimensional and the axisymmetric case is, following Cohen–Reshotko's analysis [1]:

$$\left(\frac{\partial T}{\partial y}\right)_w = T_0\,.\,S'_w\frac{\rho_w}{\rho_0}\left(\frac{m+1}{2}\,.\,\frac{U_e}{\nu_0 X}\right)^{0\cdot5} \tag{2}$$

with the assumption that $a_e = a_0$ which is justified in the vicinity of the stagnation point downstream the normal part of the shock.

The square root on the right hand side of equation (2) can be written using the transformations given in [1].

$$\begin{aligned}\left(\frac{m+1}{2}\,\frac{U_e}{\nu_0 X}\right)^{0\cdot5} &= \left(\frac{m+1}{2m}\,\frac{1}{\nu_0}\,\frac{\mathrm{d}U_e}{\mathrm{d}X}\right)^{0\cdot5}\\ &= \left(\frac{\mathrm{d}u_e}{\mathrm{d}x}\,\frac{1}{\beta\,.\,\nu_0\,.\,C}\,\frac{p_0}{p_e}\right)^{0\cdot5}\end{aligned} \tag{3}$$

where

$$\frac{2m}{m+1} = \beta$$

and

$$C = \left(\frac{T_w}{T_0}\right)^{0\cdot5}\left(\frac{T_0 + 102{\cdot}5}{T_w + 102{\cdot}5}\right).$$

The velocity gradient, $\mathrm{d}u_e/\mathrm{d}x$, which can be assumed constant [2] up to $\theta \simeq 80°$, is evaluated at the stagnation point, assuming a pressure distribution described by the modified Newtonian theory (for $M_\infty > 2$) [2] and using the incompressible form of the Bernoulli equation.

The resulting expression is [6]:

$$\frac{\mathrm{d}u_e}{\mathrm{d}x} = \frac{u_\infty}{R}\,F_1(M_\infty) \tag{4}$$

with

$$F_1(M_\infty) = \left[\frac{2}{\gamma\,.\,M_\infty^2}\,.\,\frac{T_0}{T_\infty}\left(1 - \frac{p_\infty}{p_{01}}\right)\right]^{0\cdot5}.$$

Combining and rearranging equations (1), (3) and (4) gives [6]:

$$\frac{Nu_\infty}{(Re_\infty)^{0\cdot5}} = -\frac{S'_w}{S_w}\left(\frac{C}{\beta}\right)^{0\cdot5}.\,F(\theta, M_\infty), \quad \text{for} \quad M_\infty > 2 \tag{5}$$

with

$$Re_\infty = \frac{R\,.\,u_\infty\,.\,\rho_\infty}{\mu_\infty}$$

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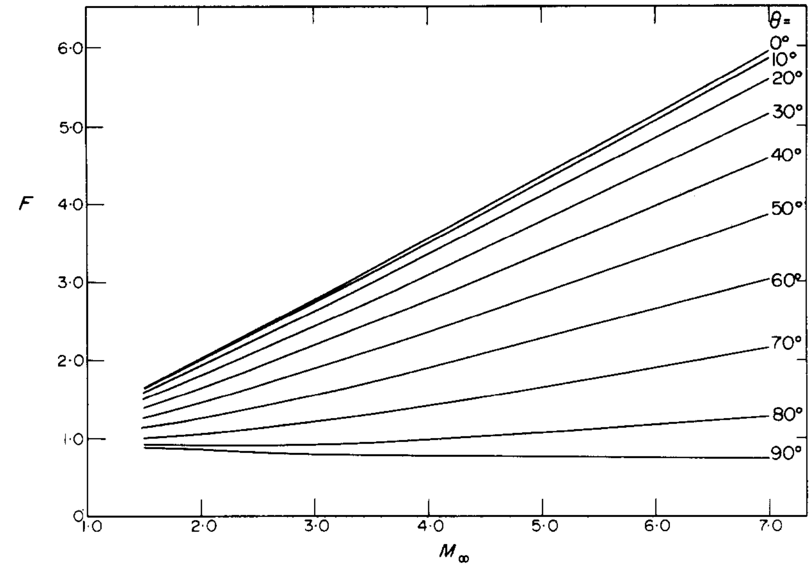

FIG. 1. Dependence of the function F on the Mach number M_∞ with θ as parameter.

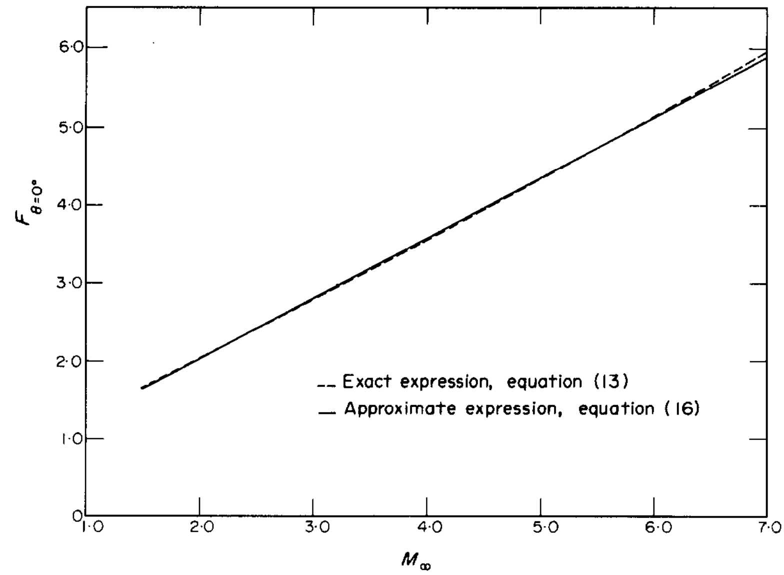

FIG. 2. Dependence of the function F on the Mach number M_∞ at the stagnation point.

and

$$F(\theta, M_\infty) = \left(\frac{p_e}{p_\infty}.F_1\right)^{0\cdot 5} = \left[\left(\left\{\frac{p_{01}}{p_\infty}-1\right\}\cos^2\theta+1\right)\left(\frac{2}{\gamma M_\infty^2}\frac{T_0}{T_\infty}\left\{1-\frac{p_\infty}{p_{01}}\right\}\right)^{0\cdot 5}\right]^{0\cdot 5} \quad (6)$$

where p_e/p_∞ is calculated from the modified Newtonian theory.

For the calculation of the ratio S_w'/S_w following the analysis of [1] and also in equation (1) it was assumed that the Prandtl number was equal to unity. To correct the calculation for gases with Prandtl numbers different from unity the following approximate relation is proposed in [3]:

$$(Nu)_{Pr} = (Pr)^{0\cdot 4}(Nu)_{Pr=1} \quad (7)$$

which is valid for Prandtl numbers from 0·6 to 1·0.

Hence,

$$\frac{Nu_\infty}{(Re_\infty)^{0\cdot 5}} = -\frac{S_w'}{S_w}(Pr)^{0\cdot 4}\left(\frac{C}{\beta}\right)^{0\cdot 5}.F(\theta, M_\infty), \quad \text{for} \quad M_\infty > 2. \quad (8)$$

The function $F(\theta, M_\infty)$ is plotted in Fig. 1 vs the Mach number M_∞ with θ as a parameter. Figure 2 shows that

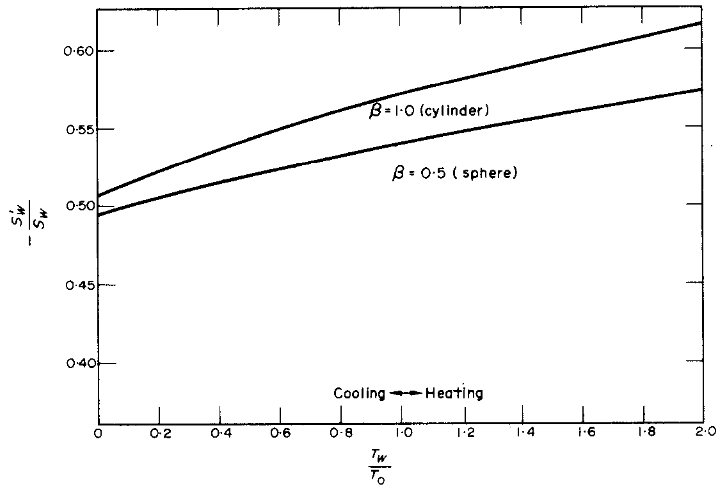

FIG. 3. Variation of S_w'/S_w with the temperature ratio T_w/T_0.

the function F for $\theta = 0°$ (stagnation point) depends nearly linearly on M_∞. F can thus be approximated by the following expression which is also shown in Fig. 2 for the Mach number range $2 < M_\infty < 7$. For

$$\theta = 0°: \quad F = 0{\cdot}48 + 0{\cdot}774 \, . \, M_\infty. \tag{9}$$

In the limiting case for $M_\infty \to \infty$ the function F at the stagnation point is given by [6]:

$$F = 0{\cdot}8931 \, . \, M_\infty. \tag{10}$$

Using the expression (9) equation (8) can be rewritten for the stagnation point heat transfer for $2 < M_\infty < 7$

$$\frac{Nu_\infty}{(Re_\infty)^{0\cdot5}} = -\frac{S_w'}{S_w}(Pr)^{0\cdot4}\left(\frac{C}{\beta}\right)^{0\cdot5}(0{\cdot}48 + 0{\cdot}774 \, . \, M_\infty). \tag{11}$$

The pressure gradient parameter β for the stagnation region is equal to 1·0 for a cylinder and 0·5 for a sphere using Mangler's [5] transformation [4].

The ratio S_w'/S_w is calculated using the analysis of [1] for $\beta = 0{\cdot}5$ and 1·0. The results are shown in Fig. 3 plotted vs the temperature ratio T_w/T_0.

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GAS ABSORPTION INTO A TURBULENT LIQUID FILM

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NOMENCLATURE

a, parameter in equation (1) $[\mathrm{T}^{-1}]$;
A_n, constants in equation (12);
B_n, constants in equation (12);
$c_{n,k}$, constants in equation (12);
C, concentration of solute $[\mathrm{ML}^{-3}]$;
C_0, inlet concentration $[\mathrm{ML}^{-3}]$;
C_s, interfacial concentration $[\mathrm{ML}^{-3}]$;
d, film thickness $[\mathrm{L}]$;
D, diffusion coefficient $[\mathrm{L}^2\mathrm{T}^{-1}]$;
h_n, constants defined by equation (18);